Azimuthal structure of charged particle emission in $^{28}$Si–Ag/Br interaction at 14.5 A GeV and $^{32}$S–Ag/Br interaction at 200 A GeV

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Presence of unusual azimuthal structures in the particle emission data obtained from the $^{28}$Si–Ag/Br interaction at 14.5 A GeV and from the $^{32}$S–Ag/Br interaction at 200 A GeV, are investigated in the framework of the Cherenkov gluon emission and/or Mach shock wave formation in nuclear/partonic medium. Nuclear photographic emulsion technique is used to collect the experimental data. The experiment is compared with the predictions of two simulations, namely (i) the Relativistic Quantum Molecular Dynamics (RQMD) and (ii) the Ultra-relativistic Quantum Molecular Dynamics (UrQMD). A charge reassignment algorithm is implemented over the outputs of the simulations to mimic the Bose–Einstein correlation (BEC) effect. Our analysis confirms presence of jet-like structures in both experiments beyond statistical noise. Such structures are more pronounced in the $^{32}$S data than in the $^{28}$Si data.

Keywords: Multiparticle production; azimuthal structure; particle correlation and fluctuation; Bose–Einstein correlation; ring- and jet-like structure.

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1. Introduction

One of the prime objectives of studying nucleus–nucleus $AB$ collisions at high-energies is to create a color deconfined state of primordial matter, the ever elusive Quark–gluon Plasma (QGP). If at all a QGP-like state is created, then a QGP
P. Mali et al.

to hadron reverse phase transition should also take place. As a result large local
densities of produced particles that are beyond trivial statistical noise, are observed
within narrow regions of phase-space.\textsuperscript{1,2} Such large particle densities in individual
events may not necessarily always be a result of a phase transition. They may also
be an outcome of an unusual behavior in some, or a systematic collective behavior in
many events. The most important reason behind particle cluster formation is the
Bose–Einstein Correlation (BEC)\textsuperscript{3–5} effect for identical mesons. Other probable
reasons are for example, formation of the QCD parton shower cascade,\textsuperscript{6} formation
of the disoriented chiral condensate,\textsuperscript{7,8} and collective phenomena like the emission
of Cherenkov gluons\textsuperscript{9,10} and/or Mach shock waves\textsuperscript{11–13} in the nuclear/partonic
medium. All of them are however mostly speculative in nature, and till date no
conclusive evidence for one or the other is found. In this paper, we are going to
investigate the presence of unusual azimuthal structures in the final state particles
using the $S$-parameter technique\textsuperscript{14} that has recently drawn the interest of several
experimental groups. As the theoretical framework of this technique is based on the
assumption of Cherenkov gluon emission or Mach shock wave formation within the
partonic/nuclear medium, we now briefly describe it. In either case the resulting
wavefront bears a conical structure, that is characterized by a semi-vertex angle $\alpha$
given by

$$\cos \alpha = \frac{v_{\text{med}}}{v} = \frac{v_0}{\mu v}. \quad (1)$$

Depending on the case as it may be, $v_{\text{med}} = v_0/\mu$ is either the velocity of the gluons
or that of the shock wave in the nuclear/partonic medium, $v_0$ is the velocity of the
 gluons or the velocity of the elastic wave in free space, $\mu$ is the refractive index of
the medium concerned and $v$ is the velocity of the partonic jet that triggers the
Cherenkov gluon emission or that of the shock wave in the nuclear medium. An
impinging nucleus is treated as a bunch of confined partons, each of which is capable
of emitting the Cherenkov gluons while traversing through a target medium. Under
favorable circumstances the conical structure of the Cherenkov wavefront may with-
stand the impact of collision, the consequence of which is reflected in the azimuthal
distribution of the final state particles.\textsuperscript{10,15} In this process if the number of emitted
gluons is large, and if each of them generates a minijet, then a ring-like structure of
final state mesons distributed over the entire target azimuth may appear. On the
other hand, for a moderate number of emitted gluons only a few jets are expected.
Similar azimuthal structures may also result due to formation of nuclear shock
waves as the impinging projectile nucleons travel with a speed greater than that of
the elastic waves through the nuclear medium. The reasons are still largely specu-
lative in nature, and the phenomenon has so far been investigated\textsuperscript{10,16–22} without
taking the BEC effect into consideration.

In the recent past, we reported an analysis on the azimuthal structure of charged
particle distribution in $AB$ collisions at 200 GeV per nucleon.\textsuperscript{23} The projectile
masses ($^{32}$S and $^{16}$O) were different, there was no multiplicity cut as regard to the
Azimuthal structure of charged particle emission

centrality of the collisions, and the experiment was compared merely with a random number-based simulation. Jet-like structures were found in the experimental data. In the present work, we report similar analyses based on the particle emission data obtained from the $^{28}$Si–Ag/Br interaction at an incident momentum of 14.5$A$ GeV, and the same from $^{32}$S–Ag/Br interaction at an incident momentum of 200$A$ GeV. The experiment is compared with more realistic simulations of $AB$ interactions, like the Relativistic Quantum Molecular Dynamics (RQMD)$^{24-26}$ and the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) models.$^{27,28}$ Moreover, using a simple charge reassignment algorithm$^{29,30}$ we also incorporate the BEC effect as a so-called “after burner” into the simulated data. The motivations are: (i) to compare experimental results on $AB$ interaction induced by almost same projectile mass but differing in $E_{\text{lab}}$ by an order, (ii) to compare the experiment with very widely used simulation data and (iii) to try to eliminate the known cause(s) of particle cluster formation, so that any discrepancy between the experiment and the simulation can be considered as a genuine signal originating from some nontrivial dynamics. For a beam energy, $E_{\text{lab}} = 14.5$ GeV equivalent nucleon–nucleon (NN) center-of-mass energy is $\sqrt{s_{\text{NN}}} = 5.39$ GeV, and the same for $E_{\text{lab}} = 200$ A GeV is 19.4 GeV. The mass numbers of the incident projectiles ($A = 28$ and 32) are very close to each other, but the $\sqrt{s_{\text{NN}}}$ values differ almost by a factor of four, while the corresponding energy densities ($\epsilon$) in the central particle producing regions, as we shall see later, differ almost by a factor of three. Peripheral interactions rarely yield any interesting feature in multiparticle production. On the other hand, central collisions where the number of participating nucleons are more, and correspondingly the number of particle producing sources are also more, one expects unusual things to happen. However, while imposing multiplicity cuts, one should be so careful that any correlated emission or systematic collective behavior of the final state particles does not get washed out by too many sources of particle production. Subsamples of central collisions are chosen by imposing stringent multiplicity cuts (to be described later) to our minimum bias event samples. The paper is organized as follows — in Sec. 2, we briefly discuss the basic features of the experimental data samples used, the methodology of analysis is described in Sec. 3, characteristics of the simulated data used to eliminate the background noise are presented in Sec. 4, the results of our analysis are discussed in Sec. 5 and in Sec. 6 we conclude with a summary on our investigation.

2. Experiment

We use Ilford G5 nuclear photo-emulsion pellicles of size 16 cm $\times$ 10 cm $\times$ 600 $\mu$m in the experiments. The pellicles are horizontally irradiated with a $^{28}$Si beam at an incident momentum of 14.5$A$ GeV from the Alternating Gradient Synchrotron (AGS) of the Brookhaven National Laboratory (BNL). Pellicles of size 18 cm $\times$ 7 cm $\times$ 600 $\mu$m are similarly irradiated with a $^{32}$S beam at an incident momentum of 200$A$ GeV from the Super Proton Synchrotron (SPS) at CERN. To
find out the primary interactions (also called event/stars), the emulsion plates are line scanned along the individual projectile tracks with Leitz microscopes under a total magnification of 300×. Koristka microscopes with a total magnification of 1500× are utilized for track counting and angle measurement. According to the emulsion terminology, the tracks emitted from an interaction are classified as (i) shower tracks, (ii) grey tracks, (iii) black tracks and (iv) the projectile fragments. Interaction with an Ag or a Br target nucleus can be ensured by choosing events for which the number of heavy tracks \( n_h = n_g + n_b \) > 8. The shower tracks caused by the singly charged particles moving with relativistic speeds, are mostly (more than 90%) due to charged mesons. Altogether, we select 158 \(^{28}\)Si–Ag/Br events with \( n_s > 50 \) and 102 \(^{32}\)S–Ag/Br events with \( n_s > 200 \) for further analysis. The average shower track multiplicities for the chosen sub-samples of events, respectively are, \( \langle n_s \rangle = 80.58 \pm 1.77 \) and \( \langle n_s \rangle = 289.67 \pm 3.12 \). In emulsion experiments, the pseudorapidity (\( \eta \)) together with the azimuthal angle (\( \varphi \)) of a track, constitutes a convenient pair of basic variables to locate a particle. An accuracy of \( \delta \eta = 0.1 \) unit and \( \delta \varphi = 1 \) mrad are achieved through the reference primary method of angle measurement. For each data set the \( \eta \) distribution can be crudely approximated by a Gaussian function, whereas \( \varphi \) is more or less uniformly distributed within 0 and \( 2\pi \). The Gaussian fit parameters are: the peak density \( \rho_0 = 29.65 \), the centroid \( \eta_0 = 1.92 \pm 0.01 \) and the width \( \sigma_\eta = 2.09 \pm 0.03 \) for the \(^{28}\)Si-sample, while \( \rho_0 = 78.2 \), \( \eta_0 = 3.26 \pm 0.02 \) and \( \sigma_\eta = 3.07 \pm 0.04 \) for the \(^{32}\)S-sample. Assuming that the projectile is completely shadowed by the target, we use the Bjorken’s formula\(^{33}\) to estimate the average energy densities in the central particle producing regions as, \( \epsilon = 0.58 \) in the \(^{28}\)Si-induced and 1.41 in the \(^{32}\)S-induced interactions. Based on the Glauber model of calculation described in Refs. 34 and 35, we also estimate the average impact parameters as \( b = 4.45 \) fm for the \(^{28}\)Si–Ag/Br sample and \( b = 2.50 \) fm, for the \(^{32}\)S–Ag/Br sample. Hence, the two subsamples considered here comprise of fairly central \( AB \) events.

3. Methodology

To search for unusual azimuthal structures, we follow the method described in Ref. 14. Without claiming any originality, we briefly describe the technique for the sake of completeness. An event with a shower track multiplicity \( n_s \), is first divided into subgroups (or clusters) of a fixed number, say \( n_d \). Each \( n_d \)-tuple of particles (tracks) are thereafter, consecutively placed along the \( \eta \)-axis. We then introduce the following quantities: (i) a size \( \Delta \eta = \eta_{\text{max}} - \eta_{\text{min}} \), (ii) a density \( \rho = n_d/\Delta \eta \) and (iii) a mean \( \eta_m = \sum_{i=1}^{n_d} \eta_i/n_d \). Here, \( \eta_{\text{max}} (\eta_{\text{min}}) \) is the largest (smallest) \( \eta \) value in the particle subgroup (cluster). Since all clusters defined by the above parameters have same number \( n_d \), they are statistically comparable with each other. Gogiberidze et al.\(^{36,37}\) used another kind of definition to explore the azimuthal structure of particle production, where instead of the cluster multiplicity, its size \( \Delta \eta \) was kept
Azimuthal structure of charged particle emission

fixed. Two other parameters expressed in terms of the azimuthal angle $\varphi$ of the shower tracks are also used to identify the jet/ring-like structures. They are:

$$S_1 = - \sum_{i=1}^{n_d} \ln(\Delta \varphi_i) \quad \text{and} \quad S_2 = \sum_{i=1}^{n_d} (\Delta \varphi_i)^2,$$

where $\Delta \varphi_i = \varphi_{i+1} - \varphi_i$ is the azimuthal gap between successive particles in the target diagram of an individual subgroup (cluster), starting from the first and second, and ending at the last and first. For simplicity, one can measure $\Delta \varphi_i$ in the unit of a full revolution ($2\pi$) of $\varphi$. In an ideal ring-like event the tracks are isotropically distributed over the whole azimuth, while in a jet-like event it would be concentrated into small but dense groups within a narrow region of $\varphi$. A schematic representation of the target azimuth of (a) an ideal ring-like and (b) an ideal jet-like structure are given in Fig. 1. The $S$-parameters and the cluster density $\rho$ will decide whether the structures are ring-like or jet-like. On the other hand, the cluster mean $\eta_m$, and the cluster size $\Delta \eta$ will help us to identify respectively, the location and the size (a measure of correlation length) of the clusters. From the definition of the $S$-parameters, it is clear that while $S_1$ is sensitive to small gaps, $S_2$ is sensitive to large gaps. In a sense $S_1$ and $S_2$ are complementary to each other. For an ideal jet-like emission $S_1 \to \infty$ and $S_2 \to 1$, while for an ideal ring-like distribution $S_1 \to n_d \ln n_d$ and $S_2 \to 1/n_d$. On the other hand, for a purely stochastic emission of particles the $\Delta \varphi$-distribution is given by

$$f(\Delta \varphi) = (n_d - 1)(1 - \Delta \varphi)^{(n_d-2)},$$

and under such circumstance the expectation values of the $S$-parameters evaluated analytically are given by,

$$\langle S_1 \rangle = n_d \sum_{k=1}^{n_d-1} \frac{1}{k} \quad \text{and} \quad \langle S_2 \rangle = \frac{2}{n_d + 1}.$$  

Distributions of $S_1$ and $S_2$ parameters would be peaked around their respective stochastic expectation values. Presence of ring-like structures are reflected as an excess in the experiment over the respective stochastic distribution in the region left to the stochastic mean. On the other hand, for jet-like structures such excesses

![Fig. 1. Schematic diagrams of an ideal (a) ring-like and (b) jet-like azimuthal structure.](image-url)
would occur in the region right to the stochastic mean. We examine the average as well as the detailed behavior of the $S$-parameters. Our task therefore, is to compare the experiment with such simulation(s) where the code(s) does (do) not include any mechanism that can lead to any unusual azimuthal structure.

4. The Simulation
A critical problem associated with the present type of analysis is to disentangle the dynamical effects that are often masked under a huge amount of trivial statistical background (noise). One way to eliminate the noise is to compare the experiment with simulated data that are based on models of $AB$ interaction. In this paper, we use two microscopic transport models namely the RQMD (version 2.4)\textsuperscript{[24–26]} and the UrQMD (version 3.3p1)\textsuperscript{[27,28]} model. These models do not incorporate any kind of particle correlation, and therefore, as far as cluster formation is concerned, they serve the purpose of generating statistical backgrounds. The rationale behind using transport models is that they treat the final freeze-out stage dynamically, do not make any equilibrium assumption, and describe the dynamics of a hadron gas-like system very well in and out of the chemical and/or thermal equilibrium. In the present experiment neither the colliding nuclei are too large, nor the collision energies involved are very high. Hence, with all probability local thermal and/or chemical equilibrium are/is not achieved. To describe such nonequilibrium many-body dynamics, transport models are a good choice.

The RQMD model is designed to give a complete description of an $AB$ collision, that starts from the initial overlapping of the colliding nuclei and ends at the final freeze-out state when strong interaction among the outgoing hadrons ceases to act. This is a semi-classical microscopic transport theory, where the incoming objects are represented by their classical trajectories and the interactions are treated stochastically. At high-energy ($E_{\text{lab}} > 10A\text{ GeV}$) $AB$ collisions, a Galuber type sequence of multiple scatterings is generated on the partonic level. Strings and resonances are excited in elementary $NN$ collisions, where the strings can overlap to form chromoelectric flux tubes called the “ropes”. Secondary particles are produced through the fragmentation of resonances, strings and ropes. The model nicely works in the BNL-AGS and CERN-SPS energy domain.

The UrQMD model\textsuperscript{[27,28]} is applicable to a wider energy range $\sqrt{s_{\text{NN}}} \sim 5–200\text{ GeV}$. The projectile and the target nuclei are treated according to a Fermi gas ansatz. In this scheme, particle production at high-energy is implemented by the color string fragmentation mechanism similar to the Lund model.\textsuperscript{[38]} The UrQMD code has been successfully used to reproduce the particle density distributions and the $p_T$ spectra of various particle species in proton–proton, proton–nucleus and $AB$ collisions.\textsuperscript{[39]} At $\sqrt{s_{\text{NN}}} \approx 10\text{ GeV}$, the model can reproduce the elliptic flow parameter $v_2$ reasonably well.\textsuperscript{[40]} However, the model does not incorporate the symmetry aspects of the fields associated with the particles, and it predicts very small Hanbury–Brown–Twiss (HBT) radii.\textsuperscript{[41,42]} It is known that the Bose–Einstein (BE)
Azimuthal structure of charged particle emission

type of identical particle effect dominates the origin of cluster formation. Due to correlated emission of like sign and/or opposite sign mesons, the particle yield with small relative momenta is enhanced, which is one of the reasons of large density of particles in the final states of any high-energy interaction. The effect is quantum statistical in nature and it cannot be simulated by transport models like the RQMD/UrQMD. Recently, a new algorithm has been developed, where the BEC is implemented by reassigning the charges of produced mesons in such a way that the overall phase-space distribution remains unaltered, the event wise multiplicities are not changed, and it looks like as if the particles (mesons) are satisfying the BE statistics. Following the BEC is numerically modeled in the form of a so-called “after burner” where the outputs of the RQMD and the UrQMD codes are used. Both the codes provide the four-coordinates as well as the four-momenta of all particles. The particle information are contained in an ASCII file written in the OSCAR format. Without changing the overall set of four-momenta, four-coordinates, or total meson charge of the system, one can generate clusters of closely spaced identical charge states of mesons by using a charge reassignment algorithm.

The RQMD code has an inbuilt provision for nuclear emulsion as a target. The default setting of the code is an equal velocity frame. Hence, we have to use a small program to convert the RQMD output to the laboratory frame. We first generate large samples of minimum bias $^{28}$Si-emulsion events at 14.5$A$ GeV and $^{32}$S-emulsion events at 200$A$ GeV, and then separate out subsamples of Ag/Br events that match in the $n_s$-multiplicity distribution with the respective experiments. The UrQMD, on the other hand, has no inbuilt provision for emulsion as a target. For each projectile (i.e., $^{28}$Si and $^{32}$S), independent event samples corresponding to Ag and Br targets are first generated. For a particular projectile, we then mix the Ag and Br events with each other. While doing so, the proportional abundances of Ag and Br nuclei in G5 emulsion are maintained. From the mixed samples, once again we select sub-samples of events in such a way as to match the respective experimental $n_s$-distribution. In both cases, the final samples of the simulated events are five times as large as the corresponding experimental one. The normalized $\eta$ and/or $\phi$ distribution(s) can be approximately described by more or less the same set of parameters as the respective experiment, Gaussian for the $\eta$ and uniform for the $\phi$. All produced mesons are retained for subsequent analysis. We then implement the charge reassignment algorithm on the mesons of both the RQMD and UrQMD generated events. Henceforth, we shall continue to call the simulated events as the RQMD + BEC and UrQMD + BEC samples.

5. Results

It is already mentioned that the $S$-parameters can be used to look for unusual structures in the target (azimuth) plane. However, in Ref. 14 it is argued that to identify a jet/ring-like structure $S_2$ is a better choice than $S_1$. As a first test, we
normalize the $S$-parameters by their respective stochastic values ($\langle S_1 \rangle$ and $\langle S_2 \rangle$) as defined in Eq. (3), and plot the histograms for two different $AB$ interactions under consideration. Fig. 2 is drawn for the $^{28}$Si-induced events ($n_d = 15$), whereas, Fig. 3 is drawn for the $^{32}$S-induced events ($n_d = 40$). For comparison the corresponding RQMD + BEC and UrQMD + BEC predictions on the $S$-parameters are also schematically presented along with the experiment. For our choice(s) of the $n_d$ values, the stochastic expectation values for the $^{28}$Si-events are $\langle S_1 \rangle \approx 48.773$ and $\langle S_2 \rangle = 0.125$, while those for the $^{32}$S-events are, respectively, $\approx 170.142$ and $\approx 0.049$. An analysis on the cluster properties similar to that of ours, was performed
by VokáI et al. in \(^{208}\text{Pb–Ag/Br}\) interaction at \(158\,\text{A}\,\text{GeV}\).\(^{20}\) They found that for the high multiplicity events the effects of any unusual azimuthal structure is almost independent of the choice of the \(n_d\) value, while for the low multiplicity events such effects diminish with increasing \(n_d\). In the present study, only high multiplicity events are chosen. We have checked that within a range, \(10 \leq n_d \leq 25\) for the \(^{28}\text{Si–interaction}\) and \(25 \leq n_d \leq 50\) for the \(^{32}\text{S–interaction}\), our results too depend only insignificantly on the choice of \(n_d\). The \(S\)-distributions of \(^{28}\text{Si–interaction}\) (Figs. 2(a) and 2(b)) are slightly left skewed about the respective stochastic mean values. As expected, we find that the RQMD + BEC and the UrQMD + BEC distributions are marginally different from their experimental counterparts. In each case, the differences between the experiment over the simulation is shown in the respective diagram with the help of shaded histograms drawn around the \(S\)-axis. We notice that there exist small experimental excesses in the left to the stochastic mean, i.e., to the jet side. The differences with experiment are larger for the RQMD + BEC simulation. For the \(^{32}\text{S–Ag/Br}\) interaction, the \(S\)-distributions once again are slightly left skewed. The skewness however, is less in this case than those of the \(^{28}\text{Si}\) distributions. For \(^{32}\text{S–Ag/Br}\) interaction the difference between experiment and simulation, once again shown by shaded histograms, lack any definite pattern and their magnitudes are smaller than the \(^{28}\text{Si}\) case. Beyond statistical uncertainties such differences are of little significance. It is to be remembered that an experimental excess in \(S_i/\langle S_i \rangle < 1\) \((>1)\): \(i = 1, 2\) region indicates ring (jet)-like structures. Based on the \(S\)-parameter distributions, we can say that in the \(^{32}\text{S}\) data hardly there is any indication of any unusual structure. However, there is a small signal of ring-like structures in the \(^{28}\text{Si}\) data. In a similar analysis of the \(^{208}\text{Pb–Ag/Br}\) data at \(158\,\text{A}\,\text{GeV}\) and \(^{197}\text{Au–Ag/Br}\) data at \(11.6\,\text{A}\,\text{GeV}\)\(^{20,21}\) experimental excess over the FRITIOF code\(^{44}\) was obtained in the \(S_2\)-distributions on either side of \(S_2/\langle S_2 \rangle = 1\). It is to be remembered that for overlapping \(\eta\)-intervals there will be strong correlations between particles belonging to different sub-groups, and this will certainly influence the statistical errors. One way to estimate the statistical uncertainties in such cases is to generate several independent sets of data based on random numbers that are similar in size, multiplicity, \(\eta\) and \(\varphi\) distributions as the experiment. One can then determine the dispersion or the standard deviation of the parameter/quantity under consideration over the number of such generated data sets. The statistical errors obtained in this way can be made free of the influence of the correlation. It may also be mentioned that the problem of \(\gamma\)-conversion and the resulting \(e^+e^-\) tracks getting mixed up with the pion tracks can influence our observation. However, this effect is more acute in vertically exposed emulsion chambers. In horizontally exposed emulsion experiments such as the present one, it is possible to follow every track back to its production point. Hence, the \(e^+e^-\) pairs arising out of \(\gamma\)-conversion (if there is any) can easily be traced back to their point of origin which will certainly be different from the primary interaction vertex. In this way, the data on shower tracks have been made free from the contamination of \(e^+e^-\) pairs.
we also study the average behavior of the $S$-parameters over a small $\eta$-interval ($\Delta \eta$). The average values are given by,

$$
S_1 = \left\langle -\sum \ln(\Delta \varphi_i) \right\rangle \quad \text{and} \quad S_2 = \left\langle (\Delta \varphi_i)^2 \right\rangle,
$$

(4)

where $\langle \rangle$ indicates event averaging. These average values are graphically presented as functions of $\Delta \eta$ in Figs. 4 and 5, respectively, for the $^{28}\text{Si–Ag/Br}$ and $^{32}\text{S–Ag/Br}$ events. In each diagram, the dashed line corresponds to the respective stochastic averages obtained from Eq. (3). The RQMD + BEC and UrQMD + BEC predictions are also incorporated in these diagrams. From these figures, it is seen that the RQMD + BEC and UrQMD + BEC predictions are systematically but consistently a little above the corresponding stochastic line, indicating a positive effect of incorporating BEC into the code. If BEC is not incorporated, then both the RQMD and the UrQMD points overlap with the stochastic lines. This feature is graphically shown at the bottom of each of the diagrams (Figs. 4 and 5), indicating thereby absence of any correlation whatsoever among the emitted mesons. The important aspect of these diagrams is that the first one or two experimental points (up to $\Delta \eta \approx 0.2–0.3$) are significantly above all other values, and beyond $\Delta \eta \approx 0.2–0.3$ the experiments are always very close to the respective simulations. In the $^{32}\text{S–Ag/Br}$ interaction the simulated results beyond $\Delta \eta \approx 0.3$ are almost

Fig. 4. Average behavior of (a) the $S_1$ parameter and (b) the $S_2$ parameter in $^{28}\text{Si–Ag/Br}$ interaction at 14.5 A GeV: $n_s > 50$ and $n_d = 15$. Horizontal dashed lines (Stochastic) are drawn following Eq. (3).
always overlapping with each other, and both are closer to the stochastic line than what they are in the $^{28}\text{Si}$–Ag/Br case. The first two or three experimental points (up to $\Delta \eta \approx 0.2–0.3$) also show significant deviation from the RQMD, RQMD + BEC, UrQMD, UrQMD + BEC and the stochastic prediction. The observation confirms that short range particle correlations other than the BE type, are present in both the experiments.

We further examine whether the contributions to the experimental excesses in the average $S$ values within a small $\Delta \eta$ ($\approx 0.1–0.3$) are coming from a large or from a small $\eta$ region. The average $S$-parameters are therefore, plotted as functions of both $\Delta \eta$ and $\eta_m$. Only the experimental distributions are shown in Figs. 6 and 7, respectively for the $^{28}\text{Si}$–Ag/Br and $^{32}\text{S}$–Ag/Br interactions. To our surprise, we notice that while the average $S_1$ values are more or less uniformly distributed over a wide $\eta$ range, there are very prominent peaks in the average $S_2$ distributions. The peaks are located within $1.0 \leq \eta_m \leq 2.0$ in the $^{28}\text{Si}$–Ag/Br and within $3.0 \leq \eta_m \leq 4.0$ in the $^{32}\text{S}$–Ag/Br interaction. Both sets of data behave similarly, and the peaks in both cases are more or less located around the central particle producing regions. Whatever may be the reason, the results suggest that to detect any unusual structure, $S_2$ is indeed a better parameter than $S_1$.

In Fig. 8 we plot the cluster density distributions for the $^{28}\text{Si}$–Ag/Br events, like before for the experiment as well as for the simulated data. Figure 8(a) represents the regions dominated by the ring-like effect $S_2/\langle S_2 \rangle < 1$ and Fig. 8(b) represents the regions dominated by the jet-like effect $S_2/\langle S_2 \rangle > 1$. Similar plots for the
\[\eta \Delta\]

\[0\]
\[0.5\]
\[1\]
\[2\]
\[3\]
\[4\]
\[5\]
\[1.5\]
\[m\]
\[\eta\]
\[0\]
\[1\]
\[2\]
\[3\]
\[4\]
\[5\]
\[\Sigma\]
\[\phi\]
\[\ln\]
\[\Delta\]
\[\eta\]
\[\eta_m\]
\[n_s > 150\]
\[n_d = 40\]

Fig. 6. Plot of (a) \(\langle -\ln(\Delta\varphi)\rangle\) and (b) \(\langle \sum(\Delta\varphi)^2 \rangle\) as a function of \(\Delta\eta\) and \(\eta_m\) in \(^{28}\text{Si–Ag/Br}\) interaction at 14.5 A GeV: \(n_s > 50\) and \(n_d = 15\).

Fig. 7. The same as Fig. 6 but for \(^{32}\text{S–Ag/Br}\) interaction at 200 A GeV: \(n_s > 200\) and \(n_d = 40\).
Fig. 8. The cluster density distributions for (a) the ring-like region, \( S_2/S_2 \langle S_2 \rangle < 1 \) and (b) the jet-like region, \( S_2/S_2 \rangle > 1 \) in \(^{28}\text{Si–Ag/Br\ interaction at 14.5\,A\,GeV: n}_s > 50 \text{ and } n_d = 15.\)

Fig. 9. The same as Fig. 8 but for \(^{32}\text{S–Ag/Br\ interaction at 200\,A\,GeV: n}_s > 200 \text{ and } n_d = 40.\)

\(^{32}\text{S–Ag/Br\ events are shown in Fig. 9. While both the }^{28}\text{Si\ diagrams are slightly left skewed, the }^{32}\text{S\ diagrams are more symmetric. If dense groups of particles are present in these data samples, then an excess experimental count over the background noise has to be found. Occasional differences between the experiment and the simulation are seen in all diagrams. In most cases in Fig. 8(a), these differences are statistically not very significant. Even in Fig. 8(a) the experimental excesses over the simulation are not too large. Differences between experiment and RQMD + BEC are more than those between the experiment and the UrQMD + BEC. The results are consistent with our previous observations.} \)
Fig. 10. The cluster size distributions for (a) the ring-like region, $S_2/\langle S_2 \rangle < 1$ and (b) the jet-like region, $S_2/\langle S_2 \rangle > 1$ in $^{28}\text{Si}$–Ag/Br interaction at 14.5 A GeV: $n_s > 50$ and $n_d = 15$.

Fig. 11. The same as Fig. 10 but for $^{32}\text{S}$–Ag/Br interaction at 200 A GeV: $n_s > 200$ and $n_d = 40$.

To have an idea about the cluster size, we plot the $\Delta\eta$ distributions in Figs. 10 and 11, respectively, for the $^{28}\text{Si}$–Ag/Br and the $^{32}\text{S}$–Ag/Br data samples. As usual, separate graphs are plotted for regions dominated by (i) the ring-like and (ii) the jet-like structures. We notice that all distributions are asymmetric (left skewed). In $^{28}\text{Si}$-induced experiment, significant excesses over the simulation are seen in the region dominated by ring-like structures ($S_2/\langle S_2 \rangle < 1$) particularly in the left to the peak (small $\Delta\eta < 0.5$) of the distribution. For $S_2/\langle S_2 \rangle > 1$, the experiment is either well-reproduced or is dominated by the UrQMD + BEC distribution. In $^{32}\text{S}$-induced events, a very narrow and sharp experimental excess is observed in the distribution at $\Delta\eta \approx 0.5$ for $S_2/\langle S_2 \rangle < 1$. While for the $^{32}\text{S}$ events in the
Azimuthal structure of charged particle emission

jet-like region \((S_2/\langle S_2 \rangle > 1)\) there is a broader and significant experimental excess over the simulation in and around the peak of the distribution \((0.5 \leq \Delta \eta \leq 0.7)\). Barring a very narrow and sharp structure around \(\Delta \eta \approx 0.5\) in Fig. 9(a), all other observations are consistent with our previous results which are, (i) mild effects due ring-like structure in the \(^{28}\text{Si–Ag/Br}\) interaction at 14.5\(\text{A GeV}\), (ii) effects due to jet-like structures in \(^{32}\text{S–Ag/Br}\) interaction at 200\(\text{A GeV}\) and (iii) differences between the experiment and RQMD + BEC are consistently larger than those between the experiment and the UrQMD + BEC.

The cluster position on the \(\eta\)-axis is investigated by plotting the \(\eta_m\)-distributions. For the \(^{28}\text{Si–Ag/Br}\) interaction, the distributions are shown in Fig. 12, and similar plots for the \(^{32}\text{S–Ag/Br}\) interactions are given in Fig. 13. The experimental distributions are more or less consistently symmetric about a mean value \(\eta_m \approx 2\) for the \(^{28}\text{Si}\) events and about \(\eta_m \approx 3.25\) for the \(^{32}\text{S}\) events. To separate azimuthal structure(s) originating due to different reasons, we use slightly more stringent conditions e.g., (i) \(S_2/\langle S_2 \rangle < 0.9\): due only to the ring-like structures, (ii) \(0.9 < S_2/\langle S_2 \rangle < 1.1\): due to the statistical effects and (iii) \(S_2/\langle S_2 \rangle > 1.1\): due only to the jet-like structures. For \(^{28}\text{Si}\) \(\eta_m\)-distribution, we see that (i) in the ring-like region \((S_2/\langle S_2 \rangle < 0.9)\) at a couple of places e.g., \(\eta_m \approx 1.5\) and 2.2, the experiment significantly exceeds the UrQMD + BEC simulation. For \(0.9 < S_2/\langle S_2 \rangle < 1.1\), the experiment and the simulation within statistical uncertainties more or less match with each other. On the other hand, for \(S_2/\langle S_2 \rangle > 1.1\) except at the extreme right-hand side tail \((\eta_m > 3.6)\) the simulation either matches or dominates over the corresponding experimental distribution. For the \(^{32}\text{S–Ag/Br}\) interaction in \(S_2/\langle S_2 \rangle < 0.9\) region small experiment-simulation mismatch are seen

![Figure 12](image-url)

Fig. 12. The cluster mean \(\eta_m\) distributions for (a) \(S_2/\langle S_2 \rangle < 0.9\), (b) \(0.9 < S_2/\langle S_2 \rangle < 1.1\) and (c) \(S_2/\langle S_2 \rangle > 1.1\) in \(^{28}\text{Si–Ag/Br}\) interaction at 14.5\(\text{A GeV}\): \(n_s > 50\) and \(n_d = 15\).
at several places. They are however, statistically not very significant. In the $0.9 < \langle S_2 \rangle / \langle S_2 \rangle < 1.1$ region there are experimental excesses over the simulation in the central $\eta_m$-region, the reason for which may probably be attributed to the limited statistics of the experiment. In the $\langle S_2 \rangle / \langle S_2 \rangle > 1.1$ region there are however significant experimental surplus over the simulation at several places, which indicate the presence of jet-like structures at different $\eta_m$-locations. We notice that in this case also the RQMD + BEC results either underestimate the experiment more, or they behave similar to the UrQMD + BEC results.

6. Summary

The azimuthal substructures of shower track emission from $^{28}$Si–Ag/Br interaction at 14.5$A$GeV and from $^{32}$S–Ag/Br interaction at 200$A$GeV are investigated with the help of the $S$-parameter technique, to check if any systematic collective behavior is present in the data samples used. The experimental results are compared with the RQMD and the UrQMD models where the BEC effect has also been taken into account as an “after burner”. In general, we find that there are occasional but significant differences between the experiment and the simulation. The RQMD model differs more than the UrQMD. We summarize below the significant points of our results obtained from the present analysis.

We conjecture that at the energy scale $E_{lab} \sim 10^1 - 10^2$ GeV per nucleon, formation of clusters is not very much energy dependent, rather the phenomenon depends more on the colliding objects.$^{14,20,21,23}$ In $^{28}$Si–Ag/Br interaction at 14.5 GeV an indication, however small it may be, of ring-like structure is observed. From the event distributions of the average values of the $S$-parameters, it transpires that a significant (60%) of $^{28}$Si-events possesses azimuthal clusters. The feature may be
attributed to a comparatively lower incident energy. Any structure if formed during the initial stage of the collision, has higher chance of surviving the impact of the collision. On the contrary in $^{32}\text{S}-\text{Ag/Br}$ interaction at $E_{\text{lab}} = 200\,\text{A GeV}$ there are indications of jet-like structures, which is a common feature at $E_{\text{lab}} = 200\,\text{A GeV}$. However, once again the event distributions of the average values of the $S$-parameters show that significant clustering in the target azimuth took place in only 25% of the events that can probably be attributed to a higher incident energy. From our analysis we can at least claim that, whatever may be the reason (nuclear or partonic) behind the signals that we see in our experiments, they are beyond a known reason like the BEC effect. In particular, the prominent short range structures in the average $S$-parameter values that we find in the central particle producing regions (Figs. 4–7), certainly reflect presence of a systematic collective behavior in multiparticle production, the exact reason for which has to be further investigated. With all probability at the incident energies concerned $E_{\text{lab}} \sim 10–200\,\text{A GeV}$, nuclear phenomenon (like for example, formation of shock waves) rather than the partonic effects, should dominate. In our $^{28}\text{Si}-\text{Ag/Br}$ data, we see small peaks at certain $\eta_m$ values in the ring-like region ($\eta_m \approx 1.5$ and 2.2). Similarly in the $^{32}\text{S}-\text{Ag/Br}$ data significant excesses in the $\eta_m$ distribution are seen in the jet region, at $\eta_m \approx 3.0$ and 4.5. This may be interpreted as the preferred $\eta$ values of jet emission. Using this information, and with the knowledge of the velocity distribution of the nucleon/partonic jet in the nuclear/partonic medium, it would be a worthwhile exercise, if we can estimate the speed of sound wave/refractive index in nuclear/partonic matter, either of which can serve significant purpose to constrain the nuclear equations of state.

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