Objectives:

- To learn to apply the ***t-test*** concerning a population mean by hand and with your calculator.
- To recognize the extensive similarities of structure and interpretation between different test procedures.
- To become familiar with the ***t-distribution*** for finding p-values.
- To discover the rolls played by sample size and sample standard deviation as well as by sample mean concerning tests of significance about a population mean.
- To understand the purpose of a ***matched pairs*** experimental design as well as the application of a ***t-test*** to the resulting data.
- To see how to check whether the *t-test* can be validly applied to a given sample of data.
- To continue to recognize the utility of graphical analyses and the limitation of summarizing a distribution of data exclusively by its mean.

Statistically significant: A result that is different enough from the expected that it is doubtful that it occurred by chance.

Test of significance: A test performed on a set of data to check to see if the difference between the expected and what occurred is statistically significant.

Null Hypothesis: What we expect, if there is no bias or influence by some other source, about a set of data, in the above example we expect heads to occur 50% of the time. Therefore when flipping a coin 100 times we expect the number of heads to occur to be 50. Symbolically we say

$$H_0 : \mu = \mu_o$$

μ is the population mean of interest, μ_o is the population mean we expect. The null hypothesis is the statement of "*no effect*" or "*no difference*" . The significance test is designed to assess the strength of evidence against the *null hypothesis.*

Alternative Hypothesis: What we expect or hope to be true. There is some influence by a source and it makes a difference (statistically significant)

Can take one of the three following forms:

(a) $H_a : \mu < \mu_o$ or (b) $H_a : \mu > \mu_o$ or (c) $H_a : \mu \neq \mu_o$

Again, μ is the population mean of interest. The alternative hypothesis states what researchers hope or suspect about the data in question. There is some influence that makes the results different than the normally expected outcome (μ_o)
The first two forms shown are ***One-sided*** the third one is ***two-sided***.

Test statistic: A value computed to standardize the difference between the null hypothesis and the value we obtained (x-bar). This is used to assess the evidence against the null hypothesis.

$$t = \frac{\bar{x} - \mu_o}{SE_x} = \frac{\bar{x} - \mu_o}{s/\sqrt{n}}$$

For the null hypothesis, we assume the null hypothesis is the true (that is μ_o is the parameter)

p-value: the probability of obtaining a value at least as extreme as the test statistic's value assuming that the Null Hypotheses (μ_o) is true. "Extreme" means in the direction of the alterative hypothesis. Therefore the p-value takes one of three forms corresponding with the appropriate H_a

$(a)\ \Pr(T_{n-1} \le t)$ area below the t - score for n - 1 degrees of freedom (df)
$(b)\ \Pr(T_{n-1} \ge t)$ area above the t - score for n - 1 df
$(c)\ 2\Pr(T_{n-1} \ge \lvert t \rvert)$ area more extreme than the t - score in both directions for n - 1 df

We judge the strength of evidence that the data provides against the null hypothesis by examining the p-value. Smaller the p-values suggest stronger evidence against the null hypothesis (the less likely a value that extreme occurred by chance alone.) and thus stronger evidence favoring the alternative hypothesis.

Typically we use the following criteria to judge the strength of evidence:

p-value > .1	Little or no evidence against H_o
.05 < p-value ≤ .10	Some evidence against H_o
.01 < p-value ≤ .05	Moderate evidence against H_o
.001 < p-value ≤ .01	Strong evidence against H_o
p-value ≤ .001	Very strong evidence against H_o

Significance level: denoted as α, is an arbitrary value set before any research begins that gives the researcher a "*cut-off"* point for the p-value. The smaller the significance level the more evidence you need to reject the null hypothesis.

Typical values for α include:
α = .10, α = .05 and α = .01

Test Decision: The decision we make on the H_o based on the p-value obtained.

If the p-value is less than or equal to the pre-chosen α the test decision is to ***reject H_o.***
If the p-value is at least the pre-chosen α then the test decision is to ***fail to reject H_o.***

IMPORTANT: Not rejecting H_o does not mean that we accept H_o or that we affirm its truth, it only means that we do not have enough evidence to contradict it.

Symbolically

If p-value ≤ α then reject H_o
If p-value > α then fail to reject H_o

It is also common to state that the result is ***statistically significant at the α level*** if our p-value is less than or equal the predetermined α

Technical Conditions: Similar to what we have seen for proportions previously

- ***Data are a SRS of the population in question***
- ***CLT rule of thumb n≥ 30 or distribution of population normal***

You should see that large samples and smaller variances in samples both cause differences in means to appear more significant – this is simply an extension what we have seen throughout the course regarding sample sizes and degree of variance.

Matched Pairs: We can analyze pairs of information by studying the difference within each pair and treating that as a its own data set. We would find the mean of the difference and the standard deviation of the differences and apply the tests to these values.