

Objectives:

- To become familiar with the idea of using **normal curves** as mathematical models for approximating certain distributions
- To develop the ability to perform calculations related to **standard distributions** with the use of a **table of standard normal probabilities**
- To discover how to use a table of standard normal probabilities to perform calculations pertaining to any normal distribution
- To learn how to use your calculator to calculate probabilities from a normal distribution.
- To assess the usefulness of a normal model by comparing its predictions to observed data.
- To develop an intuitive sense for how normal distributions relate to the questions of statistical inference.
- To acquire familiarity with judging what type of population a sample may have come from.

Symbols you need to know from this topic:

	Population	Sample
Mean	μ (mu)	$\bar{x}$ (x-bar)
Standard Deviation	σ (sigma)	s ("s")

Normal Distirbutions A family of disributions that have all of the following characteristics:

- Every Normal distribution is **symmetric**
- Every normal distribution has a **single peak** at its **center**
- Every normal distribution follows a **bell-shaped** curve

We identify each member of the family of curves by two characteristics

1. **mean**
2. **standard deviation**

μ (**mean**) : defines where its peak is located

σ (**standard deviation**) : Defines how spread out the curve is

Normal Probability Plot: A graph used to ascertain whether a set of data follows a normal pattern. This graph plots the actual observation against what you would expect to occur if the data were in fact normal. A fairly linear plot allows you to reasonably assume that the data follows a normal distribution, otherwise a normal distribution is questioned.

You can tell where on the curve **1 σ** is by where the curvature changes. *i.e.* as you follow the curve there is a point on each side of the μ where the curve goes from being curved in to curved out - the book uses the example of driving a car and it would be the point where you have to bring the steering wheel from direction to the other.

We talked briefly earlier in the semester of how the area under the curve represents a percentage of subjects - THIS IS VERY IMPORTANT TO UNDERSTAND NOW!

We also previously discussed **standardizing scores** this is also important to understand at this point!

The **standard normal distribution** is the distribution in which the $\sigma = 1.0$ and the $\mu = 0.0$. By standardizing scores we saw that we can compare SAT to ACT, we can now use the standard distribution to help us decide if a sample is part of a population or if there is something strange about our sample. If we know that a population has a certain μ we would "standardize this to $\mu = 0$, if we also knew the population had a certain σ , we would standardize this to one making it into a Z-distribution. Now when we evaluate a sample we standardize that to see where it fits on this **standard distribution**. If we are sufficiently close to 0 - the sample probably came from the

population, If it was too far from 0 we would say it probably is not part of the population - it is somehow different.

THIS MAKES NO IMPLICATION AS TO WHY. It is up to our logic, intuition and further research as to why a sample did not come from the population - IT IS NOT UP TO THE STATISTIC TO DETERMINE WHY!

How do we know how our sample relates to the population? - we use a **standard normal table** - The area under the entire *standard normal curve* is 1.00.

The table in the book gives the portion of area under the curve related to a specific Z-score, for instance if you looked up $Z=1.00$ the table gives the result of .8413, This means that 84% of the population will occur below this value, $Z = -1.00$ yields .1587, this means that almost 16% of the population will fall below this Z score, can you see the relationship here to that of the empirical rule for normal distributions?

If we need to find the number of people that are between two Z-scores - then we subtract the higher area from the lower area and this gives us the area we are looking for

What percent of people lie between $Z = -1.00$ and $Z = 1.0$?

$Z = +1.00 \Rightarrow .8413$

$Z = -1.00 \Rightarrow .1587$

Z between -1 and +1 $\Rightarrow .8413 - .1587 = 0.6826$ (or 68% look familiar?)

Percentiles: The probabilities obtained using the Standard Normal Curve may be used to report Percentiles. Using the example above a Z of 1.0 yields a percentile score of 84% (the percentage of score that are less than the given score).

Recall that the formula for standardizing scores is:

$$Z = \frac{X - \mu}{\sigma}$$

Z = Z-score

μ = mean

σ = standard deviation